OPTIMAL SYNTHESIS OF A PLANAR REACTIONLESS THREE-DEGREE-OF-FREEDOM PARALLEL MECHANISM

Jean-Francois Collard
Département de génie mécanique
Université Laval
Quebec (QC), G1V 0A6, Canada
Email: jf.collard@uclouvain.be

Clément Gosselin
Département de génie mécanique
Université Laval
Quebec (QC), G1V 0A6, Canada
Email: gosselin@gmc.ulaval.ca

ABSTRACT

A reactionless mechanism is one in which no reaction forces nor moments are transmitted to the base for any arbitrary motion. This interesting property often requires to increase the total mass and the moments of inertia, leading to reduced dynamical performances. Therefore, this paper presents an optimization approach to synthesize and improve the dynamical performance of a reactionless three-degree-of-freedom planar mechanism. The three legs of this original mechanism are composed of reactionless four-bar mechanisms dynamically balanced with only one counter-rotation at the base.

The optimization variables are the geometric and inertial parameters, while the goal is to minimize the global moment of inertia of each leg. This will reduce the power consumption of the three actuators and increase the agility. To meet physical and realistic requirements, the optimization problem is also constrained with bounds on the parameters, with the reachability of a given workspace and with a given range on a kinematic sensitivity index.

Since different initial guesses of the optimization lead to similar objective results, it is proposed to search for several local solutions (morphologies) in the design space. The final choice among these solutions is made using additional design criteria based on the sensitivity in terms of dynamic balancing and power consumption with respect to the design parameters.

INTRODUCTION

Generally, during the motion of a mechanism, forces and moments are transmitted to the base. This causes fatigue, vibration, noise, and disturbances in the supporting structure. To avoid these problems, reactionless or dynamically balanced mechanisms have been devised. For any motion of these mechanisms, there is no reaction force (excluding gravity) nor moment at its base at all times. This property is crucial especially for free-floating devices in space robotics to preserve the momentum of the moving support and also for active telescopes to reduce disturbances when moving the mirrors at high frequencies.

Dynamic balancing, also called moment balancing, involves a stationary centre of mass as well as a constant angular momentum for any configuration. If only the first condition is fulfilled, the mechanism is said to be statically balanced or force balanced. In this case, the actuators do not have to work against gravity, thereby reducing the required torques and power. The balancing of mechanisms has thus been an important research topic for several decades (e.g. see [1–3]). Extensive studies have been conducted on static and dynamic balancing of planar linkages and more recently in the context of manipulator design [4–6].

The problem of eliminating the reaction forces and moments at the base of a manipulator was addressed by a few authors, either based on constraining the trajectory planning in order to ensure the reactionless condition [7] or using additional counter-rotating inertia elements to balance the shaking moment. Nevertheless, the first approach is only suitable for some special applications. On the other hand, the addition of counter-rotations...
increases the complexity and limits the practicality [8]. It also increases the total mass and moment of inertia, reducing the dynamical performance of the mechanism.

In this paper, an original design of a planar reactionless three-degree-of-freedom parallel mechanism with three legs is first presented. Each leg is composed of a reactionless four-bar mechanism mounted on a pivot and only one counter-rotating element on the base. The multibody model is described and the dynamic balancing conditions are derived and verified with a first simulation. In the second section, the mechanism is synthesized using an optimization formulation to cope with the above-mentioned reduced performance. The objective function and all the optimization constraints are established successively. The first optimization results are described and discussed. Following this discussion, a more general optimization strategy is proposed in the third section. This strategy, called morphological synthesis, tries to explore the design space to obtain several local optima (morphologies) of the reactionless mechanism. The obtained solutions are finally discussed and a selection is proposed on the basis of balancing and dynamic performance criteria.

**REACTIONLESS MULTIBODY MODEL**

**Multibody Description**

The proposed parallel mechanism is composed of 13 bodies connected by 18 revolute joints, providing three degrees of freedom (see Fig. 1). The central moving platform is connected to the base by means of three legs oriented at 120° from each other. Each leg is composed of a four-bar mechanism whose base link is mounted on a pivot. This base link is actuated around the stationary centre of mass of the four-bar linkage with an actuator mounted on the ground link. The actuation angles are noted $\theta_i$ ($i = 1, 2, 3$). The output link of the four-bar linkage is connected to the platform and its orientation is given by angle $\varphi_i$ ($i = 1, 2, 3$). The pose vector $\mathbf{x}$ of the platform is composed of the position $(x, y)$ of the centre and of the orientation angle $\alpha$ with an offset value $\alpha_0$. All these generalized coordinates are collected in vector $\mathbf{q}$:

$$\mathbf{q} = \begin{bmatrix} \theta \varphi & \mathbf{x} \end{bmatrix}^T = \begin{bmatrix} \theta_1 \theta_2 \varphi_1 \varphi_2 \varphi_3 \alpha \end{bmatrix}^T. \quad (1)$$

It is important to note that the configuration variables of the other links of the four-bar mechanisms are not considered here since they are not useful in the optimization problem that follows, once the mechanism is dynamically balanced. However, they are obviously taken into account when simulating the dynamical behaviour of the mechanism.

The geometric design parameters of one leg are depicted in Fig. 2:

- $l_i$ is the length of link $i$ ($i = 0, 1, 2, 3$);
- $r_p$ and $r_b$ are the radii of the platform and the base respectively;
- $l_p$ is the position of the distal joint with respect to the corresponding revolute joint;
- $\alpha_0$ is the offset value of $\alpha$.

**FIGURE 1.** A REACTIONLESS THREE-DEGREE-OF-FREEDOM PLANAR PARALLEL MECHANISM WITH 9 GENERALIZED COORDINATES.

**FIGURE 2.** DESIGN PARAMETERS OF A LEG OF THE MECHANISM.
The inertial parameters considered in this planar problem are:
- \( m_i \), the mass of link \( i = 0, 1, 2, 3 \);
- \( r_i \), the position of the centre of mass of link \( i = 0, 1, 2, 3 \) along the bar (see Fig. 2). It is pointed out that the global centre of mass of the four-bar linkage is fixed on the base link and is located on the actuator joint as explained in the next Section.
- \( l_i \), the moment of inertia of link \( i = 0, 1, 2, 3 \);
- \( m_p \) and \( l_p \), respectively the mass and the moment of inertia of the platform.

The full set of 22 design parameters that is common to the three legs by symmetry is defined by vector \( \mathbf{p} \) as:
\[
\mathbf{p} = \begin{bmatrix} l_0 & l_1 & l_2 & l_3 & r_p & l_p & m_0 & m_1 & m_2 & m_3 & r_0 & r_1 & r_2 & r_3 & l_0 & l_1 & l_2 & m_p & l_p \end{bmatrix}^T.
\] (2)

The 9 generalized coordinates are constrained by a set of 6 independent assembly constraints \( \mathbf{h}(\mathbf{p}, \mathbf{q}) = 0 \) determined as:
\[
\mathbf{h} = \begin{bmatrix}
- r_b + l_b \cos \theta_1 + l_p \cos \phi_1 + r_p \cos (\alpha_0 + \alpha) - x \\
- l_b \sin \theta_1 + l_p \sin \phi_1 + r_p \sin (\alpha_0 + \alpha) - y \\
- \frac{r_0^2}{2} + l_b \cos \theta_2 + l_p \cos \phi_2 + r_p \cos (\alpha_0 + \alpha + \frac{2\pi}{3}) - x \\
- l_b \sin \theta_2 + l_p \sin \phi_2 + r_p \sin (\alpha_0 + \alpha + \frac{2\pi}{3}) - y \\
\frac{r_0^2}{2} + l_b \cos \theta_3 + l_p \cos \phi_3 + r_p \cos (\alpha_0 + \alpha - \frac{2\pi}{3}) - x \\
\frac{r_0^2}{2} + l_b \sin \theta_3 + l_p \sin \phi_3 + r_p \sin (\alpha_0 + \alpha - \frac{2\pi}{3}) - y
\end{bmatrix},
\] (3)

where \( l_b = l_0 - r_0 = l_2 - x = l_2 \frac{m_0(l_1 + l_2)}{(m_1 + m_2 + m_3 + m_p/3)} \).

Each pair of constraints represents the equality of two position vectors: the position of the tip of the distal link of a leg and the position of the corresponding joint on the platform. The solution of these assembly constraints is not unique: there are two solutions for each leg. In the following, only the two symmetrical solutions will be considered for the whole mechanism. They are represented by \( \varepsilon = +1 \) for the knee on the left and \( \varepsilon = -1 \) for the knee on the right (see Fig. 3). These two configurations will be distinguished in the following optimization process.

**Dynamic Balancing Conditions**

In order to establish the dynamic balancing conditions of the whole mechanism, each leg is balanced independently. This assumes that the central platform is divided into three independent parts, each associated with one leg. The platform can be replaced by three point masses subjected to three pairs of action-reaction forces to keep a constant distance between them as illustrated in Fig. 4. Moreover, this equivalence is valid if the total mass and the total moment of inertia are preserved. Because of the symmetry, each point mass must have a mass of \( m_p/3 \) and must be located 120° from each other, at a distance corresponding to the radius of gyration \( k_p \) of the platform [6]. Fixing each point mass to the distal link of each leg means that \( r_p \) is actually equal to \( k_p \). This leads to a first relation between three design parameters:
\[
I_p = m_p r_p^2, \tag{4}
\]

The augmented mass \( \overline{m_p} \) of the distal link becomes equal to \( m_1 + m_p/3 \). Without considering the constraint forces, this leg is statically balanced if a reactionless four-bar mechanism is chosen that rotates around its global centre of mass whose position is constant on the base link. To restrict the space requirement, the reactionless four-bar is chosen among the second family of mechanisms described in [5]. The corresponding balancing conditions established by [4] are the following:
\[
\begin{align*}
I_0 &= l_0 - l_1, \quad I_1 = l_1, \\
I_2 &= l_2 \left( 1 - \frac{m_1 r_1}{m_2 l_1} \right), \quad \overline{I}_3 = \frac{m_2 l_1 - m_1 r_1}{m_3}, \\
I_1 &= -l_2 - m_1 r_1 (l_1 + r_1), \quad \overline{I}_3 = -l_2 - \overline{m}_3 \overline{r}_3 (l_3 + \overline{r}_3), \tag{5}
\end{align*}
\]

where \( L_2 = l_2 - \frac{m_1 r_1^2}{l_1} \left( 1 - \frac{m_0 r_0^2}{l_0} \right) \) and \( \overline{I}_3 \) is the augmented moment of inertia of the distal link and the point mass fixed on it.

Furthermore, to reduce the total moment of inertia, the centre of mass \( r_0 \) of the base link and the global centre of mass \( \overline{r} \) of the other links must coincide. Using the Eqn. (5) and the expression of \( \overline{r} \) given by [5], one has:
\[
r_0 = \overline{r} = l_2 \frac{m_2 l_1 - m_1 r_1 + \overline{m}_3 l_1}{(m_1 + m_2 + \overline{m}_3) l_1}. \tag{6}
\]
design parameters collected in the following vector \( \tilde{p} \) assumed in the following, it remains a set of 13 independent conditions of Eqn. (4), (5) and (6) and assuming that the mass of the platform is a given design specification — \( m_p = 1 \text{kg} \) is assumed in the following, it remains a set of 13 independent design parameters collected in the following vector \( \tilde{p} \):

\[
\tilde{p} = [l_1 \ l_2 \ r_p \ l_p \ \alpha_0 \ m_0 \ m_1 \ m_2 \ m_3 \ r_1 \ l_1 \ l_2]^T.
\]

First Dynamical Simulation

Before optimizing the proposed mechanism, it is worth simulating it with given design parameters in order to verify the dynamical balancing conditions. It is also interesting to compute the power developed by the actuated torques to give a first idea of the power consumption. The proposed symmetrical trajectory of the platform centre is a circle of radius \( R_c \) centred in the workspace and tracked at frequency \( \nu \). The inverse dynamics is therefore simulated using the Robotran© software, developed at the Université Catholique de Louvain [9].

For this first simulation, the chosen design parameters \( \tilde{p}_0 \) are reported in Table 1 for \( R_c = 0.25 \text{m} \) and \( \nu = 0.5 \text{Hz} \). As expected, the reaction force and moment, represented in Fig. 5, have very small numerical values. These non-zero values probably come from the imperfect numerical solution of the dynamic balancing conditions and of the geometric assembly constraints during the inverse dynamic process. In Fig. 6, it is observed that the magnitude of the power of the actuator torques remains under 17.5 W. Consequently, the main goal of the following optimization is to minimize the actuator power not for a given but for an arbitrary trajectory. We propose to minimize the total moment of inertia of each leg, while maintaining the dynamic balancing property.

OPTIMIZATION AND DESIGN

In this Section, the optimization problem is stated. The objective function and all the constraints are formulated successively. Finally, the first optimization results are discussed.

Problem Formulation

The optimization problem may be stated as follows:

\[
\min_{\tilde{p}} f(\tilde{p})
\]

such that

\[
\tilde{p}_{\min} \preceq \tilde{p} \preceq \tilde{p}_{\max},
\]

\[
h(\tilde{p},q) = 0,
\]

\[
g_i(\tilde{p}) \preceq 0,
\]

and \( g_{\text{eq}}(\tilde{p},q_1,\ldots,q_{V_{\text{eq}}}) \preceq 0 \),

where:

- \( f \) is the objective function, i.e. the total moment of inertia of one independent leg;
- \( \tilde{p} \), the vector of independent design parameters, is also the vector of optimization variables;
are not useful to determine the workspace.

\( \mathbf{h}(\mathbf{p}, \mathbf{q}) \) is the vector of assembly constraints derived in Eqn. (3);

\( \mathbf{g}_i(\mathbf{p}) \) is the vector of inequality constraints ensuring feasible values for the moment of inertia of links 1 and 3;

\( \mathbf{g}_{\text{ks}}(\mathbf{p}, \alpha_1, \ldots, \alpha_{N_{\text{ks}}}) \) is the vector of inequality constraints associated with the reachability of a given workspace at \( N_{\text{ks}} \) given orientations \( \alpha_i \) of the platform;

\( \mathbf{g}_{\text{ks}}(\mathbf{p}, \mathbf{q}_1, \ldots, \mathbf{q}_{N_{\text{ks}}}) \) is the vector of inequality constraints associated with bounds on a kinematic sensitivity index evaluated at \( N_{\text{ks}} \) given points \( \mathbf{q}_i \) in the workspace.

This optimization problem is obviously not convex. It has 13 variables, one scalar objective function, 26 bound constraints, 6 non-linear equality constraints and \((2 + 2N_{\text{ws}} + 2N_{\text{ls}})\) non-linear inequality constraints as detailed below.

**Objective function** Considering the platform point mass distribution and substituting Eqn. (5) into Eqn. (7) leads us to the following simplified objective function  

\[
f = (m_2 + m_3 + m_p/3)I_2^2 - I_2 + I_0 \frac{I_2^2}{I_1} \left( \frac{(m_2 I_1 - m_1 r_1 + (m_3 + m_p/3) I_1)^2}{m_1 + m_2 + m_3 + m_p/3} + \frac{(m_1 r_1)^2}{m_2} \right). \tag{9}
\]

**Physical constraints** The first physical constraints are the lower and upper bounds on the design parameters, \( \mathbf{p}_{\text{min}} \) and \( \mathbf{p}_{\text{max}} \). Their numerical values are reported in Table 1. They should correspond to what is physically possible to build and to assemble. They will be adjusted later when a prototype will be designed.

The second set of physical constraints is related to the moment of inertia of links 1 and 3. These moments must remain positive and even greater than a minimum value. They are provided by Eqn. (5). The corresponding vector of inequality constraints is:

\[
\mathbf{g}_{\text{p}}(\mathbf{p}) = \begin{bmatrix} (I_{\text{min}} - I_1) & (I_{\text{min}} - I_3) \end{bmatrix}^T, \tag{10}
\]

where \( I_{\text{min}} \) is the minimum acceptable moment of inertia.

**Workspace constraints** While minimizing the total moment of inertia of each leg, it is important to ensure that a given workspace can still be reached. This required workspace is chosen as a disc of radius \( r_w \) centred inside the actual position workspace, while the orientation of the platform takes discrete values \( \alpha_1, \ldots, \alpha_{N_{\text{ws}}} \). For a given orientation \( \alpha_i \), the reachable workspace of the mechanism is the intersection of the reachable workspaces provided by the three legs to the platform centre (see Fig. 7). It can be noted that some links of the four-bar mechanism are not useful to determine the workspace.

In this case, the optimization problem is obviously not convex. It has 13 variables, one scalar objective function, 26 bound constraints, 6 non-linear equality constraints and \((2 + 2N_{\text{ws}} + 2N_{\text{ls}})\) non-linear inequality constraints as detailed below.

Because of the symmetry of the required and reachable position workspaces, only the first leg is considered. The corresponding reachable workspace is the area between two concentric circles of radii \( r_l \) and \( r_b \) given by:

\[
\begin{align*}
& r_l = |l_b - l_p| = \left[ I_2 \frac{m_1 (l_1 + r_1)}{(m_1 + m_2 + m_3 + m_p/3) l_1} - l_p \right], \\
& r_b = l_b + l_p = \left[ I_2 \frac{m_1 (l_1 + r_1)}{(m_1 + m_2 + m_3 + m_p/3) l_1} + l_p \right]. \tag{11}
\end{align*}
\]

The position \((x_{c,i}, y_{c,i})\) of their centre is:

\[
\begin{align*}
x_{c,i} &= r_p \cos(\alpha_0 + \alpha_i) - r_b, \\
y_{c,i} &= r_p \sin(\alpha_0 + \alpha_i). \tag{12}
\end{align*}
\]

Therefore, the required disc workspace centred in \((0, 0)\) is inside the reachable area if the two following inequality constraints hold:

\[
\begin{align*}
x_{c,i}^2 + y_{c,i}^2 &\geq (r_l + r_w)^2, \\
x_{c,i}^2 + y_{c,i}^2 &\leq (r_b - r_w)^2. \tag{13}
\end{align*}
\]

The corresponding vector of inequality constraints for the orientation \( \alpha_i \) is:

\[
\mathbf{g}_{w_{\alpha,i}} = \begin{bmatrix} (r_l + r_w)^2 - r_b^2 & -r_l^2 - 2r_b r_l \cos(\alpha_0 + \alpha_i) & \frac{r_l^2}{r_p^2} + \frac{r_b^2}{r_p^2} - 2r_b r_l \cos(\alpha_0 + \alpha_i) - (r_o - r_w)^2 \end{bmatrix}, \tag{14}
\]

concatenated in

\[
\mathbf{g}_{w_{\alpha}} = \begin{bmatrix} \mathbf{g}_{w_{\alpha,1}}^T & \ldots & \mathbf{g}_{w_{\alpha,N_{\alpha}}}^T \end{bmatrix}^T. \tag{15}
\]
Kinematic sensitivity constraints \( \hat{\mathbf{h}}(\mathbf{q}) = \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \left[ \begin{array}{c} \hat{\theta} \\ \hat{\phi} \end{array} \right] + \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \dot{\mathbf{x}} = 0 \). (20)

From this equation, the coupling matrix \( \mathbf{B}_{\phi x} \) between \( \left[ \begin{array}{c} \hat{\theta} \\ \hat{\phi} \end{array} \right] \) and \( \dot{\mathbf{x}} \) can be made explicit as:

\[
\mathbf{B}_{\phi x} = -\left( \begin{array}{c} \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \\ \frac{\partial \mathbf{h}}{\partial \mathbf{x}^T} \end{array} \right)^{-1} \frac{\partial \mathbf{h}}{\partial \mathbf{x}^T}. \tag{21}
\]

Consequently, the Jacobian matrix \( \mathbf{K} \) can be extracted from the first three rows of \( \mathbf{B}_{\phi x} \). The value of \( \sigma_p \) in the reference orientation of the platform over the required workspace is illustrated in Fig. 8. The 10 chosen positions \( \mathbf{x}_j \) of the platform are also represented in the Figure. This also shows that \( \sigma_{p,j} \) is included in the range \([1.1 \text{ m/s}, 2.1 \text{ m/s}]\) for the reference design parameters \( \mathbf{p}_0 \).

As inequality constraints, the value of \( \sigma_{p,j} \) is bounded between the lower and upper bound \( \sigma_{\max,j} \) and \( \sigma_{\min,j} \) for each position \( \mathbf{x}_j \) of the platform. The corresponding vector of inequality constraints is:

\[
\mathbf{g}_{ks} = \left[ \begin{array}{c} \sigma_{p,1} - \sigma_{\max,1} \\ \vdots \\ \sigma_{p,N_{ks}} - \sigma_{\max,j} \end{array} \right]^T \left[ \begin{array}{c} \sigma_{\min,1} - \sigma_{p,1} \\ \vdots \\ \sigma_{\min,j} - \sigma_{p,N_{ks}} \end{array} \right]^T. \tag{22}
\]

First optimization results

Before proceeding to the optimization, it is important to point out that the mass \( m_0 \) of the base link does not appear in the equations of the problem and that its moment of inertia \( I_0 \) only
The evolution of the optimization process is represented in Fig. 9. This first optimization takes 73 iterations to converge to 3.16 kgm², starting from an initial moment of inertia of 8.52 kgm². It is also interesting to observe in this Figure the evolution of the total mass of links 1, 2, and 3. Along the process, the initial total mass of 3 kg decreases first below 2 kg and increases then to reach more than 4 kg at the optimum. This means that the total mass is better distributed over the various bodies to obtain the minimum moment of inertia. All the numerical values of the optimum design parameters ˜p_{opt1} are reported in Table 1. A visual comparison between the initial and the optimum mechanisms is depicted in Fig. 12. To check the validity of this result, the optimum mechanism is simulated imposing the same circle trajectory as previously. Figure 10 shows that the mechanism is well balanced. The actuator power plotted in Fig. 11 can be compared with Fig. 6. In conclusion, this first optimization has enabled to reduce the power of the actuator from 17.5 W to 9.7 W (-45 %) with a better distribution of the mass over the bodies.

It should be pointed out that the obtained solution is a local optimum one since the MMA algorithm locally approximates the problem at each iteration. This raises the following issue : will the result be improved if another local solution is searched for? For example, a second optimization process is run from another initial guess similar to ˜p_0 except that the values of r_b and r_p have been swapped. The process takes only 31 iterations (see Fig. 9). It converges to a minimum moment of inertia of 3.29 kgm², which is only 4 % higher than the first result above! The obtained mechanism is completely different as shown in Fig. 13 but it still well balanced (see Fig. 10). Its dynamical performance is similar to those of the first one (see Fig. 11).

MORPHOLOGICAL SYNTHESIS

The last observation of the previous section shows that it is worth exploring the design space to search for other local opti-
minimum results, providing other morphologies of the mechanism. This approach developed in [12] is called morphological synthesis. Practically, this is achieved in 5 steps:

1. The whole design space is discretized into a grid. An optimization is run starting from each point of the grid.
2. All the results are filtered to remove the unfeasible solutions for which some constraints are not fulfilled;
3. The remaining solutions are grouped on the basis of the vector of their design parameters;
4. The processes that have not converged after the maximum number of iterations are carried on again, the new solutions are filtered and grouped again;
5. Finally, the selection of the best local solution relies on a more realistic design criterion.

All these steps are illustrated in the following sections.

Generation of Multiple Local Optima

Since 11 effective optimization variables are present in the optimization problem of Eqn. (8), the chosen grid is restricted to all the combinations of the bounded values of the design parameters. Because of the symmetry, each mechanism with $\varepsilon = +1$ is strictly equivalent to the same mechanism with $\varepsilon = -1$ except for the sign of $\alpha_0$. This means that the upper bound for $\alpha_0$ is not taken as $360^\circ$ but $180^\circ$ and two grids are explored for both values of $\varepsilon$. Thus, $2^{11} \times 2 = 4096$ optimization processes are run, taking more that 16 hours to be computed on a 2.53 GHz Centrino2 processor.

After the exploration of the design space by multiple optimizations, almost 25 % of the convergent and non-convergent results are unfeasible and thus eliminated. All the 3065 remaining results are then grouped into 41 solutions according to the relative distance between their vectors of design parameters which is smaller than 5 %. In the next step, 17 isolated non-convergent processes are refined,filtered and grouped again to provide finally 26 local solutions partially illustrated in Fig. 15. The obtained minimum moments of inertia go from 3.16 kgm$^2$ to 3.56 kgm$^2$ (see Table 2). The inertia of the 23$^{rd}$ solution is only 6 % larger than that of the first one. The retained solution will be selected on the basis of other criteria.
Selection Based On Parameter Sensitivity

The selection criteria are chosen as rules close to the practical design process. It is proposed to analyze the sensitivity of the obtained mechanisms with respect to manufacturing errors on one leg. The considered errors are the following:

- The position errors of the joints on each body;
- The position errors of the centre of mass of the links and the platform;
- The error on the mass of the links and the platform;
- The error on the moments of inertia of the links, the platform and the counter-rotating component.

Each position error is computed in four directions (0°, 90°, 180°, and 270°) around the nominal position as illustrated in Fig. 14. The error distance is relative to the link length \( l_i \) or the platform radius \( r_p \), with a relative error \( \rho = 1\% \). The errors on mass and moments are taken in plus or minus relatively to the nominal value with the same relative error \( \rho \). In total, 90 simulations of the errors are carried out: 12 positions of joint, 5 positions of centre of mass, 5 values of mass and 6 values of moment of inertia.

The reference trajectory is still a circle of radius 0.25 m and is tracked at 5 different frequencies from 0.1 Hz to 10 Hz. After simulating the mechanism with each error independently, three criteria are evaluated:

1. A static balancing criterion \( f_{sb} \) defined as:

\[
 f_{sb} = \max_{i=1,...,90} \left( \max_{0 \leq t \leq T} \| F_{i,t}(t) \| \right),
\]

where \( F_{i,t}(t) \) is the reaction force transmitted to the base at instant \( t \) by the mechanism subjected to the \( i^{th} \) error, and \( T \) is the period of the circle tracking. It measures the maximum loss of force balancing due to a parameter error;

2. A dynamic balancing criterion \( f_{db} \) given by:

\[
 f_{db} = \max_{i=1,...,90} \left( \max_{0 \leq t \leq T} \| Q_{i,t}(t) \| \right),
\]

where \( Q_{i,t}(t) \) is now the reaction torque transmitted to the base. It measures the maximum loss of torque balancing;

3. An actuator power criterion \( f_{ap} \) based on RMS power values over the simulation:

\[
 f_{ap} = \max_{i=1,...,90} \frac{RMS(P_i(t)) - RMS(P_0(t))}{RMS(P_0(t))},
\]

where \( P_i(t) \) is the power of one actuated torque at instant \( t \) with no manufacturing error and \( P_0(t) \) is the same power simulated with the \( i^{th} \) manufacturing error. It measures the maximum relative rise of required power due to a manufacturing error.

The evaluation of these criteria at 0.5 Hz is summarized in Table 2. It can be observed that the ratio of the standard deviation of these three criteria over their mean value is larger than it is for the objective function. This shows that the three criteria are sufficiently selective to make the final choice. Since a mean value of the three criteria is inconsistent, it is proposed to sort the mechanisms according to each criterion and to assign them the corresponding ranking. It must be pointed out that the obtained ranking is the same at all the five frequencies. Finally, the most accomplished mechanism could be reasonably selected on the basis of the mean ranking as reported in Table 3. Without weighting this mean ranking, mechanism (g) would be chosen. Nevertheless, this choice may be discussed. It should also depend on the application and other design criteria such as space requirements or manufacturing complexity. They should be taken into account, but these criteria are often difficult to determine mathematically.

**CONCLUSION**

The synthesis of a planar reactionless three-degree-of-freedom parallel mechanism has been achieved in this paper using optimization techniques. The dynamic balancing conditions have been first established and simulated. The optimal design problem has been formulated in terms of independent design parameters implicitly fulfilling the balancing conditions. Other re-
TABLE 3. BEST RANKINGS OF THE LOCAL OPTIMA BASED ON BALANCING AND DYNAMIC PERFORMANCE.

<table>
<thead>
<tr>
<th>opt. ref.</th>
<th>$f$ (kgm$^2$)</th>
<th>stat. rank</th>
<th>dyn. rank</th>
<th>power mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g)</td>
<td>3.32</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>(h)</td>
<td>3.32</td>
<td>6</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>(a)</td>
<td>3.16</td>
<td>3</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>(d)</td>
<td>3.29</td>
<td>21</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>(s)</td>
<td>3.34</td>
<td>11</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>(v)</td>
<td>3.34</td>
<td>10</td>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>(e)</td>
<td>3.29</td>
<td>20</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>(f)</td>
<td>3.30</td>
<td>22</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

FIGURE 15. EIGHT FIRST LOCALLY-OPTIMUM MECHANISMS SORTED FROM THE BEST TO THE WORST.

alistic constraints have been determined especially in terms of required workspace and kinematic sensitivity. The optimization process has first been successfully carried out. The results of a second process have shown small differences with the first optimum objective while the optimum design parameters were completely different. Therefore, an original morphological strategy has been set up to explore the design space and to search for multiple local solutions. These solutions have finally been evaluated and sorted with three design criteria based on the sensitivity of the dynamic performance to manufacturing errors.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada (NSERC) as well as the Canada Research Chair program. The authors would also like to acknowledge K. Svanberg for providing the Matlab code of the MMA algorithm.

REFERENCES