What dynamics should impedance-controlled robots render?

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Abstract— **While impedance control is the standard framework for physically interactive robots, the design choice of what dynamics should be rendered requires additional information (assumptions on environment, in-situ data). The range of dynamics which can be rendered by a robot is informed by its mechatronic design (actuators, physical compliance, inner-loop control), and these mechanical design decisions must be made in advance. How can a mechatronic design be evaluated when the system objectives and environment dynamics are not quantified? This paper presents performance metrics proposed for pHRI in literature, and seeks to move towards a unified methodology for mechatronic design on interactive robots: supporting potential performance and safety over a set of environments.**

I. INTRODUCTION

Impedance control is a well-established framework for interactive robots, separating the robot from a human or environment at a power-continuous interaction port [1], [2]. This re-frames mechatronic design from improving metrics established for closed systems, such as reference tracking and disturbance rejection, to design of the dynamics rendered at the interaction port. Coupled stability can be concluded given the robot's dynamics and a set of environment dynamics (e.g. passive, pure inertia, or pure stiffness). Evaluating performance in advance is more difficult, both in motivating a quantitative objective and evaluating the metric.

As the rendered dynamics can be reshaped with control, it may be tempting to claim the optimal dynamics can be found in-situ. However, the range of dynamics rendered over all controllers is limited, and informed by the mechatronic design (including actuators, joint stiffness, end-effector compliance, and inner-loop control) by reasons both theoretical (Bode's integral theorem, bandwidth limitations) and practical (sampling time, stability limits). The mechatronic design should allow the desired dynamics to be rendered, but those dynamics may be unknown in advance. To address this problem for physical Human-Robot Interaction (pHRI), general mechatronic design goals have converged to low impedance robots: lightweight, compliant mechanisms [3], broadly motivated to reduce potential risk in collision [4].

This paper presents performance objectives which have been proposed for pHRI applications, and frames mechatronic design as supporting performance over a set of possible objectives. Many applications of pHRI can be framed as allowing an operator to induce their desired state to the robot. Broad motivations for low impedance mechanical design are recovered, but are considered alongside unintended drawbacks in several application examples.

II. PERFORMANCE METRICS

PHRI is often motivated by tasks which cannot be completely automated due to task variation, sensing limitations, or flexible manufacturing. More generally, the desired robot behavior cannot be readily expressed as a controller policy and reference trajectory defined from available sensors alone. One solution is that the human brings observation of both the robot state and in-situ desired state (reflecting the current configuration of the task), manipulating the robot into the appropriate configuration. This category of pHRI includes collaborative lifting [5], kinematic demonstrations for lead/walk-through teaching, hands-on-payload manipulation, and collaborative assembly [6]. Different metrics proposed for evaluating the system performance in this form of pHRI are introduced in this section.

Note that there are also performance metrics for the isolated robot, namely the accuracy and range of dynamics which can be rendered. Examples include Z-width [7], M-width [8], and robust impedance control performance [9]–[11]. This section is concerned with performance metrics which consider coupled task performance.

A. Performance as transparency

The common application of pHRI - hands-on manipulation of a robot - is that which makes minimal assumptions about the task and human. Here, the task is posed as a goal position or trajectory which is known to the operator but not the robot. The operator realizes a motor control policy in contact with the robot, possibly based on an internal model of how the robot will respond, which causes the robot to converge to the desired position or trajectory.

If the operator is modelled as an impedance, and the robot as an admittance, a possible objective is minimizing the total absolute energy transferred between the robot and human in the realization of a desired velocity profile $v^d(t)$. With the robot and human coupled with a shared velocity $v(t)$, and with equal and opposite forces $f(t)$, this metric is:

$$\begin{aligned} \int_0^\infty \left| f^T(t) v^d(t) \right| dt &= \|f^T(t) v^d(t)\|_1 & (1) \\ &\le \|f(t)\|_2 \|v^d(t)\|_2 & (2) \end{aligned}$$

which is given by Hölder's inequality [24]. Perseval's theorem can be used to rewrite these quantities in the frequency domain [25], where $F(\omega) = \mathcal{F}\{f(t)\}$, the unilateral Fourier

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Description	Adapted parameters	Task model?	Human model?	Reference
Critical damping	B		varying stiffness, online	[12], [13]
Minimize tracking error	K	a priori trajectory		[14]–[16]
Minimize jerk	$K,\ B$			[17]
Minimize interaction forces	M			[18]
Task certainty	$f_r\,(t)$	learned probabilistic model	PD controller	[19]
Operator metabolic cost	(mode parameter)			[20]
Operator fatigue	$F,\ f_r\,(t)$		a priori fatigue model	[21]
Operator force range	K			[22]
Proxy for operator intent	B			[23]

TABLE I: Design objectives in literature used to adjust outer-loop impedance/admittance controllers. Parameters shown are those which are equivalent to an admittance model of $f_h = M\ddot{x} + B\dot{x} + Kx + f_r\,(t)$, where $f_r\,(t)$ is a feed-forward robot force trajectory, f_h the human force, and M, B, K mass, damping, and stiffness.

transform of $f(t)$, and the system dynamics in frequency domain $V^d(\omega) = A(\omega)F(\omega)$ to give:

$$\int_0^\infty \left| f^T(t)v^d(t) \right| dt \quad \leq \quad \|V^d(\omega)\|_2 \left\| \frac{V^d(\omega)}{A(\omega)} \right\|_2 \qquad (3)$$

Although $v^d(t)$ (and therefore $V^d(\omega)$) are unknown, it is known that human force output rolls off at a maximum of 8 Hz [26]. Under the assumption that any slack in the inequality (3) is monotonic in $|A(\omega)|$, the optimal admittance can be found via

$$\begin{aligned} \arg \min_{A(\omega)\in\mathcal{A}_s} \int_0^\infty \left| f^T(t)v^d(t) \right| dt &= \arg \min_{A(\omega)\in\mathcal{A}_s} \frac{\|V^d(\omega)\|_2^2}{\|A(\omega)\|_2} \\ &= \max_{A(\omega)\in\mathcal{A}_s} \|A(\omega)\|_2 \qquad (4) \end{aligned}$$

where $\mathcal{A}_s$ denotes the set of admittances which can be safely rendered by the robot.

The robot dynamics which minimizes absolute energy transfer over an arbitrary input is the maximum admittance (or, minimum impedance), recovering the common objective of minimizing damping and inertia of a robot being manipulated in free-space. As the bandwidth of the human input is limited, this objective is not substantially impacted by the high-frequency dynamics of the robot - such as the resonance from a compliant human interface.

B. In-situ performance

With more specific knowledge of a task, an appropriate objective can be defined. These objectives are typically either functions of measured quantities (position trajectories, operator metabolic cost) or based on the evaluation of simple models (human as time-varying stiffness, probabilistic learned task). Established metrics from literature are reviewed in Table I.

These objectives often require in-situ data, due to the limitations in a priori models, although many approaches use some assumptions on either the task or human behavior to motivate an objective. The approaches in Table I exclusively tune impedance/admittance control parameters towards their objectives, often staying within the impedance parameter safety bounds provided by the hardware manufacturer. The adaptation is often motivated to give personalized performance of the device, adapt to changes in the task or task mode, or simply improve performance of a metric which cannot be predicted.

C. Generalizing performance

To what degree can performance metrics in interactive systems be cast into a unified framework? Safety for interactive systems has a unfied framework: guarantee coupled stability over a set of environmental dynamics. Closed (i.e. non-interactive) control systems have design methods which accommodate general frequency-domain or state-space performance metrics. The current range of proposed interactive metrics suggests that different applications lend themselves to unique objectives over the robot states, informed by the intuition of the application engineer.

III. RELATING MECHATRONIC DESIGN TO PERFORMANCE OBJECTIVES

While impedance control parameters can be adapted in situ, the physical dynamics of the robot limit the range of dynamics which can be rendered with various impedance controllers. This is due to both direct reasons in reshaping the dynamics, and the limitations in impedance parameters which are imposed by stability conditions.

A. Limitations in reshaping dynamics

Reshaping dynamics with feedback control suffers from limitations of inner-loop control (position and force, for admittance and impedance control, respectively), often meaning that only the low-frequency dynamics of a system can be modified. Although the inner-loop bandwidth changes with the environmental contact conditions, this rule of thumb that high-frequency dynamics cannot be reshaped is often useful. For example, at high-frequencies, compliance through control is not possible - and for collision with high stiffness environments (high-frequency excitation), physical compliance becomes critical. Further limitations of reshaping the rendered dynamics through control are presented below.

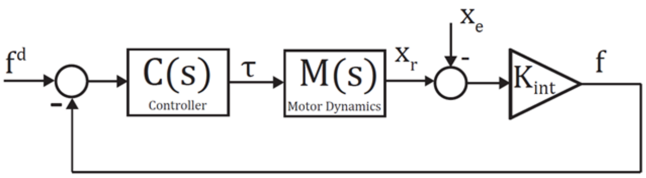

Fig. 1: Impedance control structure for demonstrating the limitations in reshaping the rendered dynamics $x_e \rightarrow f$

1) Admittance control: Under admittance control, well-established results for a single-inertia, backdriveable system showed the limitations in reducing inertia while maintaining passivity under time delay [27].

2) Impedance control: Bode's integral theorem can show the limitations of causal feedback control for impedance control. Let robot dynamics $M(\omega)$, spring interface K, force control law $C(\omega)$, and environment position x_e and robot position x_r as seen in Figure 1, for any causal controller $C(\omega)$ the following quantity is conserved by Bode's integral theorem [25]:

$$\frac{F(\omega)}{X_e(\omega)} = \frac{K}{1+KM(\omega)C(\omega)} \qquad (5)$$

$$\int_0^\infty \ln\left|\frac{F(\omega)}{X_e(\omega)}\right| = -\frac{\pi}{2}\lim_{s\rightarrow 0} K_{int}C(s)M(s). \qquad (6)$$

If $M(s) = (Is^2 + Bs)^{-1}$

$$\int_0^\infty \ln\left|\frac{F(\omega)}{X_e(\omega)}\right| = -\frac{\pi K_{int}}{2I}. \qquad (7)$$

This conserved quantity shows that while control can reshape the dynamics, the intrinsic dynamics of the robot set limits in what rendered dynamics are feasible. Often called the water-bed effect, it shows that reducing the magnitude of the response at some frequencies requires increasing it at others.

B. Safety principles

For many interactive robots; the range of dynamics which can be rendered (e.g. $\mathcal{A}_s$ in (3)) is also limited by safety [28]; either in maintaining passivity at the interaction port under time delay [27] or model uncertainty [29]. Alternative approaches (seeking to avoid the conservatism of passivity) consider the coupled stability of the system for a specific class of environment (pure stiffness, pure inertia) [10], [30], [31]. Mechatronic design which aims to relax safety conditions is established [18], and other work adapts the impedance parameters when instability is detected [32].

While passivity is the traditional approach for coupled stability of general interactive robots, it is rarely necessary for robots which only interact with humans. The conservatism of passivity is well-established [33], and stability conclusions with humans have also been established by assuming the human as an uncertain inertia [31]. The conservatism of passivity in human interaction tasks can be understood by replacing passivity with a mixed passivity/small-gain coupled stability condition, where the low impedance of a human allows the coupled system to meet a small-gain condition at high-frequencies, tolerating more passivity violations [29]. A high-stiffness environment (high impedance) does not allow such relaxation.

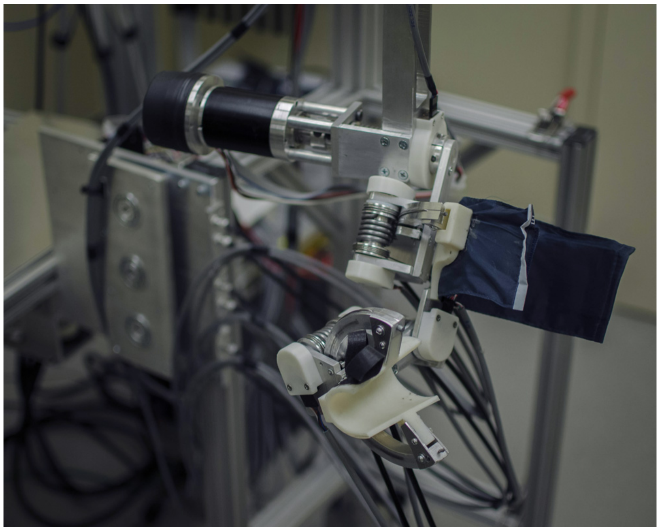

Fig. 2: Exoskeleton system with series-elastic actuators for five DOF (first vertical DOF above image), and remote actuation for the three distal DOF by bowden cables

However, interaction with a high-impedance environment still occurs in pHRI, especially when the human is guiding the robot into contact with a high-stiffness environment. The importance of stiffness in contact stability [34] is established, and a tradeoff between force feedback gain, environmental stiffness and sampling time in theory [2] and practice [30]. If this contact with the environment will occur at an uncertain location, often the damping of the free-space guidance must be increased throughout the workspace, to ensure the coupled stability condition is met when contact occurs.

IV. LIMITATIONS OF COMPLIANT OR LIGHTWEIGHT ROBOTS

Many robots designed for human-robot-environment interaction, such as where an operator guides the robot into contact with a high-stiffness environment, can simply include compliance where the environment is contacted as seen in Figure 3. This reduces peak force in collision and relaxes coupled stability conditions, often without major compromise to the system performance.

On the other hand, systems which require substantial control authority over the payload, such as in Figure 4, compliance is not an option. A lower stiffness coupling with the stone saw gives (for a fixed mass) a higher-magnitude resonance peak to the position response of the stone saw from external forces. A stiff coupling allows the high intrinsic stiffness of the robot to push the resonance peak to higher frequencies, where the inertia reduces the magnitude. Design for this application is demanding: a high-stiffness environment, with substantial noise injection from

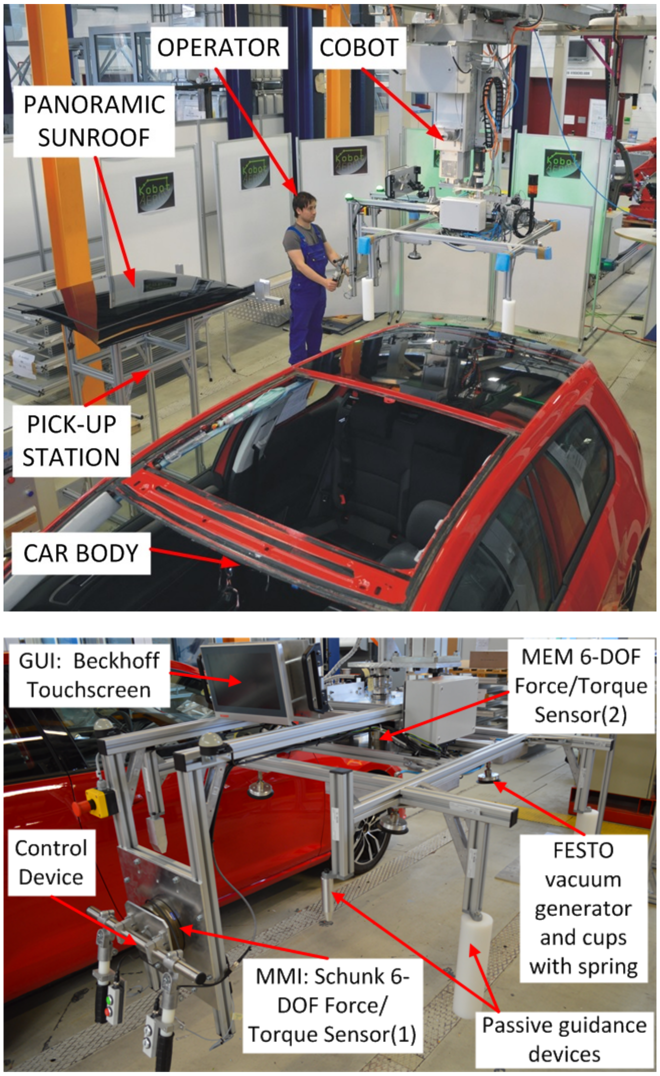

Fig. 3: Cobot handling system for the installation of glass panels on cars, where substantial compliance is included at the interface with the car

Fig. 4: Interactive robot for stone carving with a demonstration cut in progress below

the vibration of the stone saw and cutting forces. A robust controller design methodology allows design of an admittance controller which is stable in contact, and constraints in performance (transparency for the human operator) must be accepted.

Just as compliance must be considered in context, reductions in system inertia do not universally improve task performance. Some approaches to reducing inertia in serial manipulator systems use remote actuation via cable systems or bowden-sheath cable systems. These drivetrains can introduce substantial friction, although the impact of this can be reduced through control if there is joint-side sensing (i.e. joint angle measurements at the actual joint, such as seen in Figure 2). However, bowden-sheath systems can also exert a translational force through spring effects from the bending of the sheath, applying a torque at all joints proximal of the bowden-actuated joint. This force can be difficult to model, making it difficult to compensate in a feed-forward manner. This force cannot be distinguished from the human or environment force, effectively reducing the accuracy of the achieved dynamics. While the rendered inertia is reduced, in the case of exoskeleton actuation this high-frequency change in dynamics is not as critical as the low-frequency error introduced by the unmodelled cable force in this application.

V. CONCLUSION

General design goals of low inertia and low stiffness are well-established for interactive robots, but a specific application may have a more natural objective. A mathematical statement of objective offers many advantages (in-situ adaption/learning, advanced control techniques), but incomplete models (i.e. limited models of the environment) limit the application of these techniques. A more general framework of defining the objective function over a set of possible objectives is, although conceptually pleasing, difficult mathematically. Assuming that the environment/human input is in some way reflecting the goals of the system requires further assumptions about human goal-oriented and motor-control behavior. A generalized framework for deciding dynamics a priori is difficult but provides

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